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Analysis of Throughput-Constrained Tactical Wireless Networks

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***Abstract*—Communications networks supporting tactical military operations are often constrained by satellite throughput capacities. Using network architecture information and a traffic flow model based on packet captures from a large field exercise, we formulate and solve a multicommodity maximum flow problem (MCMFP) to approximate communications traffic within a tactical wireless network. The model provides a deterministic method of quickly identifying and analyzing network bottlenecks and quantifying the value of additional network throughput. We use this model to examine the interplay between a logical traffic model and the physical architecture used to support it. We also present a method for optimizing the performance of a network by re-arranging existing wireless links. To our knowledge, we are the first to use an MCMFP to identify network bottlenecks within a U.S. Marine Corps tactical wireless network using a traffic model based on actual packet captures.**

I. Introduction

Military forces are increasingly reliant on the rapid, reliable transfer of digital data throughout the battlefield in order to conduct command and control (C2) [1]–[3]. This data can include voice and text-based communications, intelligence reports, photographs, video feeds, C2 system-specific traffic, and other Internet traffic. The ability to exchange data in a digital communications network is often limited by a relatively small number of network bottlenecks [4]. *Tactical* military networks are particularly susceptible, as they often rely on wireless satellite and terrestrial radio links that have much smaller capacities than physical transmission media such as twisted-pair copper or fiber optic cable. Network planners may increase the capacity of links or add redundant links, but this can be technically challenging and expensive [5], [6], and demand for satellite bandwidth is far outstripping supply [7].

Low-fidelity analytic methods, including mathematical network optimization, are often used to quickly characterize communications network performance without the expense and technical challenge of conducting high-fidelity simulation or field testing [8], [9]. These methods generally ignore many aspects of actual communications networks, including delay, collisions, and protocol interactions, but can be useful in capturing important aspects of the performance of a communications system and verifying the results of higher-fidelity simulations [10], [11].

The *multicommodity maximum flow problem* (*MCMFP*) maximizes delivered flow across a network subject to constraints on arc capacity [12]. The problem considers multiple, distinct flows or *commodities*, often distinguished based on flow source and destination [13]. This multi-source, multi-destination approach can roughly approximate the exchange of communications traffic [10], [12], [14].

Bapeswara Rao *et al.* [15] and [16] analyze the maximum flow in a communications network, respectively developing algorithms based on the binary form of natural numbers and a fuzzy matrix approach. Frank [17] uses a maximum flow formulation to examine the survivability of C2 networks. Based on the MCMFP of [18], Alderson *et al.* [19] examine performance tradeoffs in Enhanced Position Location Reporting System (EPLRS) networks, and [20] and [21] determine good positions for access points in wireless mesh networks. Murray *et al.* [22] use network optimization techniques to examine the protection of critical infrastructure within a telecommunications network. Huang and Fang [23] use a maximum flow formulation to analyze a network with switched beam directional antennas, and [24] use a minimum-cost maximum-flow formulation to identify network bottlenecks.

In previous work, we consider the use of mobile ad-hoc network (MANET) technology to support tactical U.S. Marine Corps communications at the battalion level and below using various network architectures [25] and in an electromagnetic spectrum-constrained environment [26]. This research complements and extends that work by considering satellite and terrestrial multiplexed communications above the battalion level. Using network architecture information and a traffic flow model based on packet captures from a large U.S. Marine Corps field exercise, we explore the interplay of network infrastructure and network traffic. Our mathematical model provides a deterministic method of quickly identifying and analyzing network bottlenecks and quantifying the value of additional network throughput. We also present a method for optimizing the performance of a network by re-arranging the locations of existing wireless links. To our knowledge, we are the first to use an MCMFP to identify network bottlenecks within a U.S. Marine Corps tactical wireless network using a traffic model based on actual packet captures.

This paper is organized as follows. In Section II, we describe our MCMFP formulation to model the flow of communications traffic within a tactical network. In Section III, we describe our network topology and traffic flow model using statistics and visualizations. In Section IV, we present the results of analyses using several variations of our formulation. In Section V, we conclude with a brief description of the second phase of this research, where we will use the results of our deterministic methods to inform development and verification of network architectures built in OPNET [27].

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II. Problem Formulation

We wish to explore network bottlenecks that occur when one or more wireless communications links have reached their maximum operating capacity. We formulate and solve an MCMFP to push as much traffic as possible to support a given traffic model. We model the wireless wide area network (WAN) supporting a Marine Corps task force as a *directed graph* G of nodes $i \in N$ (alias j) and arcs $(i, j) \in A$. Nodes represent both routers and radios, including ground-based terrestrial radios and satellite communications (SATCOM) radios. Nodes may be collocated at physical *sites*; we assume nodes at a particular site are directly connected via fiber optic cable or other high-throughput media that does not constrain network performance. Every node may serve as a relay; router nodes may also serve as sources and destinations for communications traffic (i.e., we do not explicitly represent terminal devices). Directed arcs $(i, j) \in A$ represent wireless connections between nodes. While our formulation allows directed arcs between every pair of nodes (i.e., a *complete* graph), in practice the network we consider is quite *sparse*.

A commodity in this multicommodity network flow model represents a traffic flow between a given source and destination over a period of time, a practice consistent with the Internet Engineering Task Force [28] and network optimization literature [12]. Let $D \subseteq N$ be the set of nodes that serve as destinations for network traffic, indexed by $d = 1, 2, \ldots, |D|$, and let $(i, d) \in P$ be the set of *source-destination pairs* that must communicate. Let the decision variable F_{ij}^d indicate the amount of flow from i to j destined for d. A flow need not be symmetric or have an associated return flow; in this way we assume user datagram protocol (UDP)-like traffic transmission without handshake dialogues [29]. Based on packet captures and subject matter expertise, we assign each source-destination pair a positive value $\textit{weight}_i^d$ to indicate the relative importance of the associated flow. Let the decision variable S_i^d indicate the total amount of flow successfully sent from i to d by any directed path(s) through the network.

Following [18] and [30], we characterize the performance of our network using an objective function that aims to maximize overall delivered flow. Our formulation values only flows between source-destination pairs, so there is no incentive to provide flows between any other pairs of nodes. Consider a *linear weighted sum* objective function, i.e.:

$$\max_{F,S} \sum_{(i,d)\in P} \textit{weight}_i^d S_i^d. \tag{1}$$

While easy to understand and explain, this objective function does not incentivize the equal distribution of flow among source-destination pairs. That is, it may (and does in practice) maximize the flow between some pairs at the expense of providing no flow between other pairs. However, the *log-utility* objective function we use:

$$\max_{F,S} \sum_{(i,d)\in P} \log_2 \left(\textit{weight}_i^d S_i^d \right) \tag{2}$$

provides decreasing benefit for increasing flow, thus incentivizing the equal distribution of flow among pairs. Additionally, flows less than one receive a penalty, so there is strong incentive to provide at least unit flow between each pair.

We use the following sets of constraints to ensure balance of flow at each node:

$$\sum_{i:(j,i)\in A} F_{ji}^d - \sum_{i:(i,j)\in A} F_{ij}^d = \left\{ \begin{array}{ll} S_j^d, & j \neq d \\ -\sum\limits_{i,i\neq d} S_i^d, & j = d \end{array} \right\} \quad \forall j \in N, d \in D. \tag{3}$$

That is, the total amount of flow delivered to a destination node d from a source node j is equal to the difference between incoming and outgoing flows destined for d at each j.

In our formulation, the maximum throughput of the network is limited only by the capacities of the individual wireless connections. We limit the aggregate wireless data transmission rate on each arc (regardless of traffic source or destination) using a positive value $\textit{capacity}_{ij}$. If no other capacity is available, each arc is thus constrained:

$$\sum_{d\in D} F_{ij}^d \leq \textit{capacity}_{ij} \quad \forall (i,j) \in A. \tag{4}$$

Following [18] and [20], we assume nodes themselves do not have limits on capacity.

In a variation of our formulation, each arc may also have an associated decision variable E_{ij}, which indicates the maximum amount of extra flow that can be added to each arc. These extra flows are constrained by a positive value $\textit{extraCapacity}_{ij}$, representing the maximum possible data rate of a radio:

$$E_{ij} \leq \textit{extraCapacity}_{ij} \quad \forall (i,j) \in A. \tag{5}$$

Replacing (4), the associated alternate constraint on arc capacity is:

$$\sum_{d\in D} F_{ij}^d \leq \textit{capacity}_{ij} + E_{ij} \quad \forall (i,j) \in A. \tag{6}$$

The total amount of extra flow in the network is limited by a positive value *maxExtraCapacity*:

$$\sum_{(i,j)\in A} E_{ij} \leq \textit{maxExtraCapacity}. \tag{7}$$

To ensure non-negativity, we include the constraints:

$$S_i^d \geq 0 \quad \forall i \in N, d \in D \tag{8}$$
$$F_{ij}^d \geq 0 \quad \forall (i,j) \in A, d \in D \tag{9}$$
$$E_{ij} \geq 0 \quad \forall (i,j) \in A. \tag{10}$$

Our original MCMFP (i.e., not allowing extra flow) aims to maximize the total value of delivered flow (2), subject to constraints (3)-(4) and (8)-(9). Our variation that allows extra flow (*MCMFP-E*) comprises (2)-(3) and (5)-(10). The nonlinear objective function (2) is concave and strictly increasing and all constraints are linear, so these are concave nonlinear maximization problems. Note our formulations do not consider network queuing delays, collisions, or wireless transmission losses, and thus provide an upper bound on the throughput capacity of a given WAN.

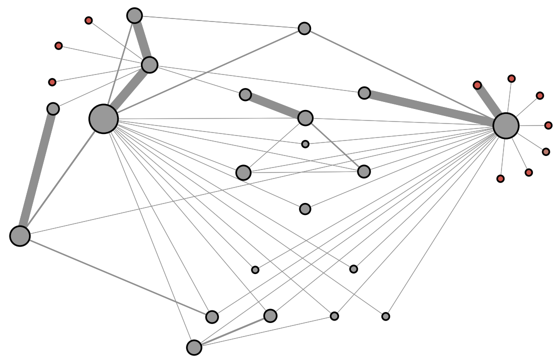

Fig. 1. Physical WAN topology depicted as a graph of sites (circles) connected by links. Line width is proportional to relative link capacity; circle size is proportional to relative betweenness centrality of the associated site. Red circles indicate sites with no redundant links into the network.

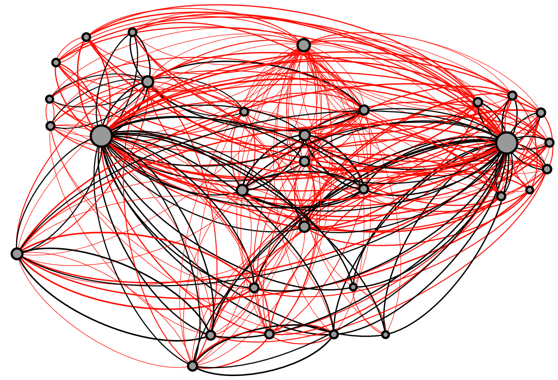

Fig. 2. Logical traffic model indicating the source and destination of all traffic flows. Line width is proportional to relative $weight_{id}$ value; circle size is proportional to relative aggregate weighted out-degree of the associated site. Red indicates flows that must traverse relay nodes en route to their destination.

III. Network Description

A. Network Topology

We consider the wireless WAN supporting a Marine Expeditionary Force (MEF) during a large field exercise aboard Camp Pendleton and the Marine Corps Air Ground Combat Center, California, and Marine Corps Air Station Yuma, Arizona. A MEF is the largest U.S. Marine Corps operational force, comprising 46 000-90 000 Marines, and is capable of conducting a wide range of military operations, including combined arms operations involving ground troops, tanks, artillery, and close air support [2]. The MEF WAN we consider interconnects the MEF headquarters with units at the regiment and battalion level. In practice, data connectivity is extended below the battalion level using tactical MANETs and both wired and wireless local area networks (LANs), but these systems are typically not interconnected below the battalion level and are excluded from this WAN-centric analysis. See [25] for a related analysis within the Marine infantry battalion.

The MEF WAN is modeled as a directed graph of 78 nodes and 218 directed arcs. These arcs range in capacity from 512 kbps to 54 Mbps. We aggregate nodes into 31 physical sites consisting of collocated nodes. For clarity in our visualizations, we present aggregate wireless capacity between sites using 112 directed *links*; the sites and links thus represent the underlying *multigraph* [31]. This representation of the network is depicted in Fig. 1 (note relative position in the figure is not related to actual real-world physical location). The relative aggregate capacity of each link is indicated by the width of each line. Links that are noticeably wider comprise arcs representing terrestrial ground-based radios capable of much greater throughput rates than satellite radio links. The size of each circle depicts the relative *betweenness centrality*, a measure of importance based on the number of times a site is included in shortest paths between sites [32]. The two largest circles represent the MEF main and forward headquarters. The sole connection to the Internet and global information grid is provided via *Standardized Tactical Entry Points* (*STEPs*) [33], represented as the circle at the top-center of the figure.

Van Mieghem [34] describes measures and statistics that can be useful in characterizing the functionality and performance of a communications network. The *density* of the WAN, indicating the extent to which sites are directly linked to other sites and calculated as:

$$density = \frac{|A|}{|N|(|N|-1)} \tag{11}$$

is only 0.116, where a complete graph has a value of 1. In general, such sparse graphs have higher *directed path lengths* (i.e., number of hops) between nodes. Indeed, the average directed path length for our WAN is 2.19, and the longest path (i.e., the *network diameter*) is 4. This indicates that if all nodes must communicate, many must rely on wireless transmission systems and routers at other sites to deliver traffic to the correct destination. In a battlefield environment, these hops present vulnerabilities to transmission jamming and physical destruction. This problem is exacerbated by low *redundancy*: ten sites (the red circles in Fig. 1) have only one link into the WAN and can thus be easily disconnected.

B. Network Traffic Model

Based on packet captures from a major field exercise, subject matter expertise, and previous work [25], we develop a hierarchical traffic model where most traffic is exchanged between units that are immediately adjacent within the chain of command. We use $weight_i^d$ values from one to ten to indicate the relative importance of desired flows. In general, flows between senior and subordinate units have higher importance than flows between units at the same hierarchical level. Fig. 2 shows the 286 desired traffic flows between source-destination pairs $(i,d) \in P$ within the WAN traffic model. The width of each line indicates the relative $weight_i^d$ of the flow, and the size of each circle indicates the relative *out-degree* of the associated site, where the weighting for each node is $\sum_{d \in D} weight_i^d$. Red indicates flows which must traverse at least one hop; black indicates flows that can (assuming unlimited $capacity_{ij}$) pass directly from source to destination. Over 70% (202) must hop through at least one relay node, a characteristic common of military networks [8]. Most traffic remains within the MEF; about 21% (60) are directed to and from the STEP site (the top-center circle) and generally have relatively low $weight_i^d$ values.

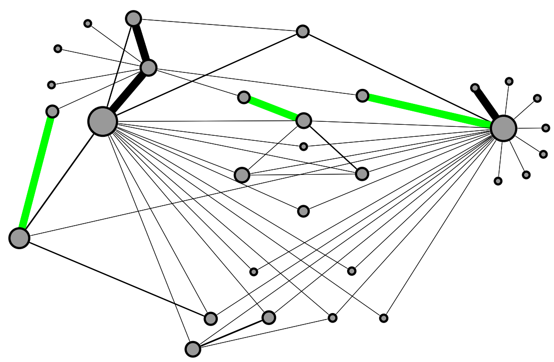

Fig. 3. Illustration of the solution to the MCMFP. Line width is proportional to the relative amount of aggregate flow going through the link. Black indicates bottlenecks, i.e., links with no remaining capacity. The size of each circle is proportional to the number of unique source-destination flows passing through that site.

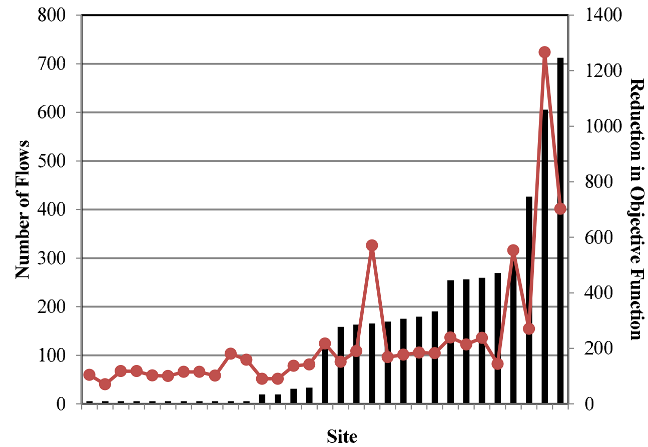

Fig. 4. Rank-ordered list of the number of flows that pass through each site (black bars), and the reduction in the objective function when that site is removed from the network (red line).

IV. Analysis

To solve our problem, we use the General Algebraic Modeling System (GAMS) [35] and the Couenne optimizer [36]. We use Microsoft VBA, Python, Networkx [37], and Gephi [38] to analyze and visualize the network and the outputs of our optimization methods. We find the ability to converge to the global optimum is sensitive to the range of input values (specifically, the size of arc capacities), so we allow *dynamic scaling* to allow the solver to scale input values to manageable ranges [39], [40]. We are exploring the use of providing initial solutions in order to solve larger problem instances.

A. Identifying Bottlenecks

In our first analysis we solve our MCMFP using the traffic model depicted in Fig. 2 to determine total network flow and the locations of bottlenecks. Fig. 3 illustrates the output, where the width of each link indicates the relative amount of aggregate traffic flow between each site. The color indicates the capacity remaining on that link, where black is a link with no remaining capacity, i.e., all arcs between the sites are saturated and thus a bottleneck is formed.

The optimality conditions of a maximum flow problem guarantee there is no *residual network capacity* between any source-destination pair [12], i.e., it is not possible to push more flow between a source and destination. Thus at optimality, arcs with remaining capacity cannot directly connect a source-destination pair, as there would be incentive to push (and capacity to support) additional flow. The links with remaining capacity (depicted as green lines in Fig. 3) connect high-throughput, relatively inexpensive ground-based radios. To increase network capacity, these ground-based radio links should be installed to directly connect source-destination pairs. Otherwise, these links may have unused capacity while expensive, low-capacity satellite links are saturated. Essentially, these underutilized links indicate an inefficient mismatch between the required logical traffic flows (i.e., Fig. 2) and the physical infrastructure to support those flows (i.e., Fig. 1).

To get a sense of site centrality and importance, we next sum the total number of unique source-destination flows through each site. The size of each circle in Fig. 3 is relative to the total number of unique source-destination flows that pass through the associated site. In Fig. 4, these counts are rank-ordered and displayed as bars associated with the left vertical axis. Over a third of WAN sites (11 of 31) relay traffic only for themselves and one other node, while two sites relay over 29% of traffic. These two sites (the largest circles in Fig. 3 and the right-most bars in Fig. 4) are the MEF main and forward headquarters; they serve as hubs for a large amount of network traffic flow. Their centrality presents a potential vulnerability, as their removal (whether due to equipment failure or enemy action) would greatly reduce total delivered traffic.

To obtain a measure of the impact of site removal, we re-run our optimization problem and selectively remove each of the sites by zeroing all of their inbound and outbound arc $\mathit{capacity}_{ij}$ values. The red line in Fig. 4 (associated with the right vertical axis) illustrates the reduction in the objective function (2) when each site is removed. While in general the more valuable sites are those with more flows transiting through them, there are important exceptions. We find the most important sites are those that serve as hubs for sites that have no alternate path into the rest of the network, i.e., those sites the removal of which creates disconnected sub-networks. Network flow optimization – as opposed to simply counting links or flows – brings to light the interplay of the physical WAN infrastructure and the logical traffic model.

B. Adding Network Capacity

The identification of bottlenecks alone does not inform us how best to relieve congestion. To obtain this, we determine the optimal locations within the network for additional capacity by using our MCMFP-E formulation. We first allow a total $\mathit{maxExtraCapacity} = 100$ Mbps to be divided among any number of existing arcs. The results are displayed in Fig. 5. Over 30% of the additional throughput is allocated to the satellite radio links connecting the MEF to the STEP site. These arcs are visible as the noticeably thicker lines connected to the top-center point in Fig. 5. The size of each circle indicates the relative weighted out-degree of extra capacity.

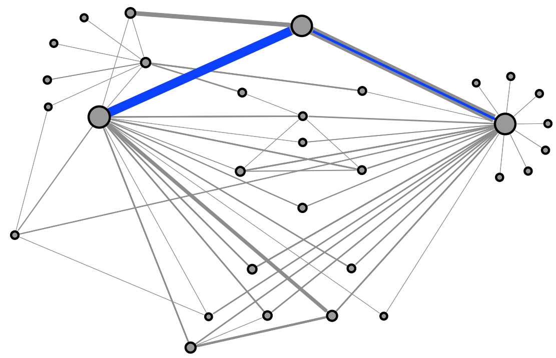

Fig. 5. Optimal locations for an additional 100 Mbps of network capacity. Line width is proportional to the relative amount of extra capacity; circle size is proportional to the relative weighted out-degree of these extra capacities. Blue indicates the two most valuable locations for extra capacity.

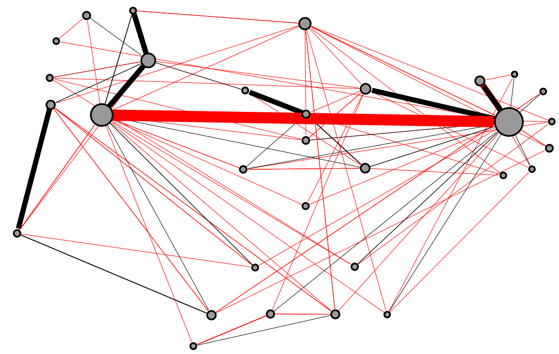

Fig. 6. Optimal WAN topology using rearranged original network arcs, increasing total delivered flow by 9.5%. Line width is proportional to relative link capacity; circle size is proportional to relative betweenness centrality of the associated site. Red indicates arcs that moved from Fig. 1.

In reality, the number of links that can be assigned additional capacity is likely to be quite small. To model this, we further modify MCMFP-E by constraining the total number of links that can receive additional flow. Let the binary variable $Y_{ij} \in \{0, 1\}$ indicate whether arc (i, j) receives extra flow. We add the following constraints:

$$mY_{ij} \geq E_{ij} \quad \forall (i, j) \in A \tag{12}$$

$$\sum_{(i,j) \in A} Y_{ij} \leq \mathit{maxExtraArcs} \tag{13}$$

where m is a large positive constant. Constraints (12) ensure Y_{ij} is one if there is flow on (i, j). Constraints (13) ensure there are no more than *maxExtraArcs* arcs receiving additional capacity. We solve this mixed-integer nonlinear program using Couenne, setting $\mathit{maxExtraArcs} = 2$. We find that network performance can be most improved by increasing flow along the uplinks from the forward and main MEF headquarters to the STEP site (indicated in blue in Fig. 5). Unfortunately, these SATCOM links are often expensive and scarce [1], [4]. Reducing the dependence on these links (from the perspective of our model, reducing the value weight_i^d of flows to the STEP site) can also improve network performance according to (2).

C. Optimizing Network Topology

In our final analysis, we wish to determine the optimal network topology to support the traffic model depicted in Fig. 2 by rearranging the given arcs. That is, we allow the optimization solver to take the existing arcs (and their associated capacities) and rearrange them between nodes to maximize the total value of delivered flow (2). We modify our original MCMFP as follows. Let $w \in W$ be the set of wireless arcs with capacities specified by $\mathit{capacity}_{ij}$. Let the binary decision variable $Z_{wij} \in \{0, 1\}$ indicate whether arc w connects nodes i and j. We replace constraints (4) with:

$$\sum_{d \in D} F_{ij}^d \leq \sum_{w} \mathit{capacity}_w Z_{wij} \quad \forall (i, j) \in A \tag{14}$$

and add the following constraints:

$$\sum_{w} Z_{wij} \leq 1 \quad \forall (i, j) \in A \tag{15}$$

$$\sum_{ij} Z_{wij} \leq 1 \quad \forall w \in W \tag{16}$$

where (15) ensures each arc $(i, j) \in A$ has no more than one wireless arc w and (16) ensures each wireless arc w is assigned no more than once.

This combinatorial optimization problem is a variation of the NP-complete *network design problem* [41], [42]. We are unable to find solutions using Couenne, so we replace our nonlinear objective function (2) with (1) (thus creating a mixed-integer linear program) and solve using CPLEX-Distributed [43]. Even this linear version of the problem takes 48 hours to solve on a distributed cluster of 14 high-end desktop computers. The resultant network is depicted in Fig. 6, where the width of each line is relative to the total capacity of the link, and red indicates a moved link (69% are moved). Using these arc assignments Z_{wij}, we re-solve our original MCMFP problem. The optimized WAN topology increases overall delivered flow $\sum_{(i,d) \in P} S_i^d$ by 9.5%, and increases the total value of delivered flow (2) by 3.6%. An unexpected benefit of this new topology is that there are no sites lacking redundant links.

V. Conclusion and Future Work

Our low-fidelity techniques provide a relatively quick method of identifying and analyzing network bottlenecks without resorting to time-consuming, high-fidelity simulation. We identify inefficient mismatches between desired traffic flows and network infrastructure. To the extent possible, high-capacity links provided by ground-based terrestrial radios should connect sources and destinations with high expected traffic flow. We acknowledge, however, that real-world factors will limit network topology design options. Network bottlenecks can be relieved by storing more information locally or in a distributed manner [8]. We find that the most valuable place for additional network capacity is on the links connecting the MEF to the STEP site, even though the traffic flows passing over these links have relatively small weights.

We briefly present an optimization method to improve network performance by merely rearranging existing wireless arcs. Though the associated combinatorial optimization problem can be time-consuming to solve, it may be worth the effort for architectures that are not expected to immediately change.

In follow-on work, we will use OPNET [27] to conduct high-fidelity, stochastic network simulation to explicitly consider the queuing aspects of *real-time traffic management* [10] that we ignore in the present work. Future research could examine electromagnetic and other specific network disruptions (accidental or intentional) on the design and operation of wireless communications backbones (e.g., [44]–[46]).

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